Introduction to cell-centered Lagrangian schemes

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Eulerian formalism (spatial description)

- fixed referential attached to the observer
- fixed observation zone through the fluid flows

Lagrangian formalism (material description)

- moving referential attached to the material
- observation zone moved and deformed as the fluid flows

Lagrangian formalism advantages

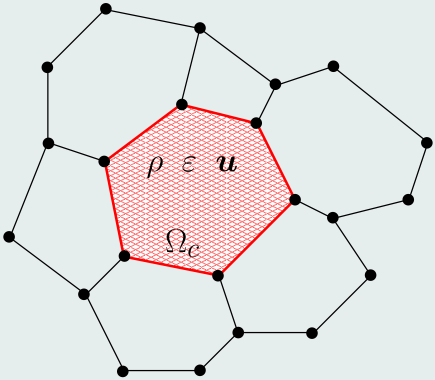
- adapted to problems undergoing large deformations
- naturally tracks interfaces in multi-material flows
- avoids the numerical diffusion of the convection terms

Lagrangian formalism drawbacks

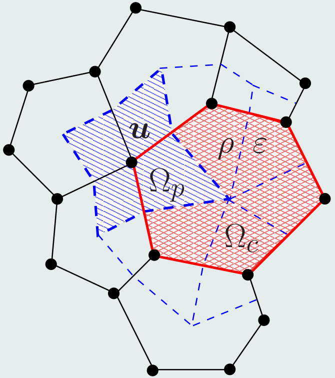
- **Robustness issue in the case of strong vorticity or shear flows**

$\Longrightarrow$ ALE method (Arbitrary Lagrangian-Eulerian)

Cell-centered formulation



Staggered formulation



Definitions

- ρ the fluid density
- $\boldsymbol{u}$ the fluid velocity
- e the fluid specific total energy
- p the fluid pressure
- $\varepsilon = e - \frac{1}{2}\boldsymbol{u}^2$ the fluid specific internal energy

Euler equations

- $\dfrac{\partial \rho}{\partial t} + \nabla_x \cdot \rho\, \boldsymbol{u} = 0$
- $\dfrac{\partial \rho\, \boldsymbol{u}}{\partial t} + \nabla_x \cdot (\rho\, \boldsymbol{u} \otimes \boldsymbol{u} + p\, \mathrm{I}_d) = \boldsymbol{0}$
- $\dfrac{\partial \rho\, e}{\partial t} + \nabla_x \cdot (\rho\, \boldsymbol{u}\, e + p\, \boldsymbol{u}) = 0$

Thermodynamical closure

- $p = p(\rho, \varepsilon)$ Equation of state

Moving referential

- $\boldsymbol{X}$ is the position of a point of the fluid in its initial configuration
- $\boldsymbol{x}(\boldsymbol{X}, t)$ is the actual position of this point, moved by the fluid flow

Trajectory equation

- $\dfrac{\partial\, \boldsymbol{x}(\boldsymbol{X}, t)}{\partial t} = \boldsymbol{u}(\boldsymbol{x}(\boldsymbol{X}, t), t)$
- $\boldsymbol{x}(\boldsymbol{X}, 0) = \boldsymbol{X}$

Material derivative

- $f(\boldsymbol{x}, t)$ is a smooth fluid variable
- $\dfrac{\mathrm{d}\, f(\boldsymbol{x}, t)}{\mathrm{d}t} = \dfrac{\partial\, f(\boldsymbol{x}, t)}{\partial t} + \boldsymbol{u} \,.\, \nabla_x f(\boldsymbol{x}, t)$

Definitions

- $\tau = \frac{1}{\rho}$ the specific volume
- $\mathrm{U} = (\tau, \boldsymbol{u}, e)^{\mathrm{t}}$ the solution vector
- $\mathrm{F(U)} = (-\boldsymbol{u},\, \mathbb{1}(1)\,p,\, \mathbb{1}(2)\,p,\, \mathbb{1}(3)\,p,\, p\,\boldsymbol{u})^{\mathrm{t}}$ where $\mathbb{1}(i) = (\delta_{i1},\, \delta_{i2},\, \delta_{i3})^{\mathrm{t}}$
- $a = a(\rho, \varepsilon)$ the sound speed

Updated Lagrangian formulation

- $\rho \dfrac{\mathrm{d}\,\mathrm{U}}{\mathrm{d}t} + \nabla_x \,.\, \mathrm{F(U)} = 0$ Moving configuration

Non-conservative formulation

- $\rho \dfrac{\mathrm{d}\,\mathrm{U}}{\mathrm{d}t} + \mathrm{A}_x(\mathrm{U}) \dfrac{\partial\,\mathrm{U}}{\partial x} + \mathrm{A}_y(\mathrm{U}) \dfrac{\partial\,\mathrm{U}}{\partial y} + \mathrm{A}_z(\mathrm{U}) \dfrac{\partial\,\mathrm{U}}{\partial z} = 0$
- $\mathrm{A}_n = \mathrm{A}_x\, n_x + \mathrm{A}_y\, n_y + \mathrm{A}_z\, n_z$ with $\boldsymbol{n}$ a unit vector
- $\lambda(\mathrm{U}) = \{-\rho\, a,\, 0,\, \rho\, a\}$ the eigenvalues of $\mathrm{A}_n(\mathrm{U})$

Deformation gradient tensor

- $\mathsf{J} = \nabla_X \boldsymbol{x}$ — Jacobian of the fluid flow
- $|\mathsf{J}| = \det \mathsf{J} > 0$ — Positive control volume
- $\nabla_X \cdot \left(|\mathsf{J}|\mathsf{J}^{-t}\right) = \mathbf{0}$ — Piola compatibility condition

Mass conservation

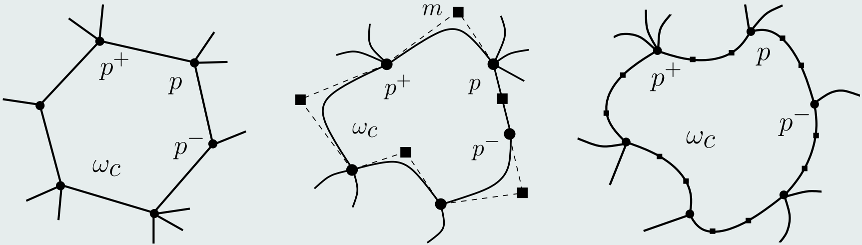
- $\int_{\omega(0)} \rho^0 \,\mathrm{d}V = \int_{\omega(t)} \rho \,\mathrm{d}v$
- $\int_{\omega(t)} \rho \,\mathrm{d}v = \int_{\omega(0)} \rho \,|\mathsf{J}| \,\mathrm{d}V$
- $\rho \,|\mathsf{J}| = \rho^0$

Total Lagrangian formulation

- $\rho^0 \dfrac{\mathrm{d}\,\mathsf{U}}{\mathrm{d}t} + \nabla_X \cdot \left(|\mathsf{J}|\mathsf{J}^{-1}\mathsf{F}(\mathsf{U})\right) = 0$ — Fixed configuration

Définitions

- $0 = t^0 < t^1 < \cdots < t^N = T$ a partition of the time domain $[0, T]$
- $\omega^0 = \bigcup_{c=1,I} \omega_c^0$ a partition of the initial domain ω^0
- ω_c^n the image of ω_c^0 at time t^n through the fluid flow
- m_c the constant mass of cell ω_c
- $\mathrm{U}_c^n = (\tau_c^n, u_c^n, e_c^n)^{\mathrm{t}}$ the discrete solution



(a) Straight line edges (b) Conical edges (c) Polynomial edges

Figure: Generic polygonal cell

Integration

- $\mathrm{U}_c^{n+1} = \mathrm{U}_c^n - \dfrac{\Delta t^n}{m_c} \displaystyle\int_{\partial \omega_c} \overline{\mathrm{F}} \,.\, \boldsymbol{n} \,\mathrm{d}s$
- Integration of the cell boundary term (analytically, quadrature, ...)

General first-order finite volumes scheme

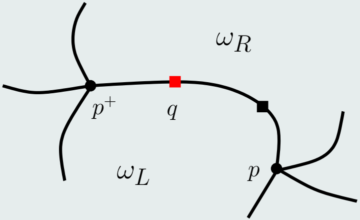
- $\mathrm{U}_c^{n+1} = \mathrm{U}_c^n - \dfrac{\Delta t^n}{m_c} \displaystyle\sum_{q \in \mathcal{Q}_c} \overline{\mathrm{F}}_{qc} \,.\, l_{qc} \boldsymbol{n}_{qc}$
- $\overline{\mathrm{F}}_{qc} = (-\overline{\boldsymbol{u}}_q,\ \mathbb{1}(1)\,\overline{p}_{qc},\ \mathbb{1}(2)\,\overline{p}_{qc},\ \overline{p}_{qc}\,\overline{\boldsymbol{u}}_q)^{\mathrm{t}}$ numarical flux at point q
- $\boldsymbol{x}_q^{n+1} = \boldsymbol{x}_q^n + \Delta t^n\, \overline{\boldsymbol{u}}_q$

Definitions

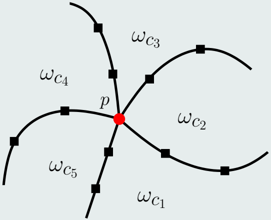
- $\mathcal{Q}_c$ the chosen control point set of cell ω_c
- $l_{qc}\boldsymbol{n}_{qc}$ some normals to be defined

Remark

- $\overline{\mathsf{F}}_{qc}$ is local to the cell ω_c
- Only $\overline{\boldsymbol{u}}_{qc} = \overline{\boldsymbol{u}}_q$ needs to be continuous, to advect the mesh
- Loss of the scheme conservation?



(a) Face control point



(b) Grid node

Figure: Points neighboring cell sets

1D numerical fluxes

- $\overline{p}_{qc} = p_c^n - \widetilde{z}_{qc}\,(\overline{\boldsymbol{u}}_q - \boldsymbol{u}_c^n)\,.\,\boldsymbol{n}_{qc}$
- $\widetilde{z}_{qc} > 0$ local approximation of the acoustic impedance

Conservation

- $\sum_c m_c \, \mathrm{U}_c^{n+1} = \sum_c m_c \, \mathrm{U}_c^n + \mathrm{BC} \quad ?$
- For sake of simplicity, we consider $\mathrm{BC} = 0$
- Necessary condition: $\sum_c \sum_{q \in \mathcal{Q}_c} \overline{p}_{qc} \, l_{qc} \boldsymbol{n}_{qc} = \mathbf{0}$

Example of a solver: LCCDG schemes

- Conditions suffisantes
- $\forall p \in \mathcal{P}(\omega), \quad \sum_{c \in \mathcal{C}_p} \left[\overline{p}_{pc}^- \, l_{pc}^- \boldsymbol{n}_{pc}^- + \overline{p}_{pc}^+ \, l_{pc}^+ \boldsymbol{n}_{pc}^+ \right] = \mathbf{0}$

$$\Longrightarrow \quad \overline{\boldsymbol{u}}_p = \Big(\sum_{c \in \mathcal{C}_p} \mathrm{M}_{pc} \Big)^{-1} \sum_{c \in \mathcal{C}_p} \Big(\mathrm{M}_{pc} \boldsymbol{u}_c^n + p_c^n \, l_{pc} \boldsymbol{n}_{pc} \Big)$$

- $\forall q \in \mathcal{Q}(\omega) \setminus \mathcal{P}(\omega), \quad (\overline{p}_{qL} - \overline{p}_{qR}) \, l_{qL} \boldsymbol{n}_{qL} = \mathbf{0} \quad \Longleftrightarrow \quad \overline{p}_{qL} = \overline{p}_{qR}$

$$\Longrightarrow \quad \overline{\boldsymbol{u}}_q = \left(\frac{\widetilde{z}_{qL} \, \boldsymbol{u}_L^n + \widetilde{z}_{qR} \, \boldsymbol{u}_R^n}{\widetilde{z}_{qL} + \widetilde{z}_{qR}} \right) - \frac{p_R^n - p_L^n}{\widetilde{z}_{qL} + \widetilde{z}_{qR}} \, \boldsymbol{n}_{qf_{pp^+}}$$

Convex combinaison

- $$\mathrm{U}_c^{n+1} = \mathrm{U}_c^n - \frac{\Delta t^n}{m_c} \sum_{q \in \mathcal{Q}_c} \overline{\mathrm{F}}_{qc} \cdot l_{qc} \boldsymbol{n}_{qc} + \frac{\Delta t^n}{m_c} \mathrm{F}(\mathrm{U}_c^n) \cdot \underbrace{\sum_{q \in \mathcal{Q}_c} l_{qc} \boldsymbol{n}_{qc}}_{=\mathbf{0}}$$
- $$\mathrm{U}_c^{n+1} = (1 - \lambda_c)\, \mathrm{U}_c^n + \sum_{q \in \mathcal{Q}_c} \lambda_{qc} \overline{\mathrm{U}}_{qc}$$

Definitions

- $\lambda_{qc} = \frac{\Delta t^n}{m_c} \widetilde{z}_{qc}\, l_{qc}$ and $\lambda_c = \sum_{q \in \mathcal{Q}_c} \lambda_{qc}$
- $$\overline{\mathrm{U}}_{qc} = \mathrm{U}_c^n - \frac{\left(\overline{\mathrm{F}}_{qc} - \mathrm{F}(\mathrm{U}_c^n)\right)}{\widetilde{z}_{qc}} \cdot \boldsymbol{n}_{qc}$$

CFL condition

- $$\Delta t^n \leq \frac{m_c}{\sum_{q \in \mathcal{Q}_c} \widetilde{z}_{qc}\, l_{qc}} \quad \left(= \frac{|\omega_c^n|}{a_c^n \sum_{q \in \mathcal{Q}_c} l_{qc}} \qquad \text{if} \quad \widetilde{z}_{qc} \equiv z_c^n = \rho_c^n\, a_c^n \right)$$

Semi-discret first-order scheme

- $m_c \dfrac{\mathrm{d}\,\mathsf{U}_c}{\mathrm{d}t} = - \sum_{q \in \mathcal{Q}_c} \overline{\mathsf{F}}_{qc} \,.\, l_{qc} \boldsymbol{n}_{qc}$

Gibbs identity

- $T\,\mathrm{d}S = \mathrm{d}\varepsilon + p\,\mathrm{d}\tau = \mathrm{d}e - \boldsymbol{u} \,.\, \mathrm{d}\boldsymbol{u} + p\,\mathrm{d}\tau$

Semi-discret production of entropy

- $m_c\, T_c \dfrac{\mathrm{d}\,S_c}{\mathrm{d}t} = m_c \dfrac{\mathrm{d}\,e_c}{\mathrm{d}t} + \boldsymbol{u}_c \,.\, m_c \dfrac{\mathrm{d}\,\boldsymbol{u}_c}{\mathrm{d}t} + p_c\, m_c \dfrac{\mathrm{d}\,\tau_c}{\mathrm{d}t}$
- $m_c\, T_c \dfrac{\mathrm{d}\,S_c}{\mathrm{d}t} = \sum_{q \in \mathcal{Q}_c} \widetilde{z}_{qc}\, l_{qc} \left[(\overline{\boldsymbol{u}}_q - \boldsymbol{u}_c) \,.\, \boldsymbol{n}_{qc} \right]^2 \geq 0$

Positivity of the discrete scheme

F. Vilar, C.-W. Shu and P.-H. Maire, *Positivity-preserving cell-centered Lagrangian schemes for multi-material compressible flows: Form first-order to high-orders. Part II: The 2D case.* JCP, 2016.

High-order extension of the finite-volume scheme

- MUSCL, (W)ENO, DG, ...

Mean values equation

- $\mathrm{U}_c^{n+1} = \mathrm{U}_c^n - \dfrac{\Delta t^n}{m_c} \sum_{q\in\mathcal{Q}_c} \overline{\mathrm{F}}_{qc} \,\boldsymbol{.}\, l_{qc}\boldsymbol{n}_{qc}$
- In $\overline{\mathrm{F}}_{qc}$, the mean values are substituted by the high-order values U_{qc} in ω_c at points q

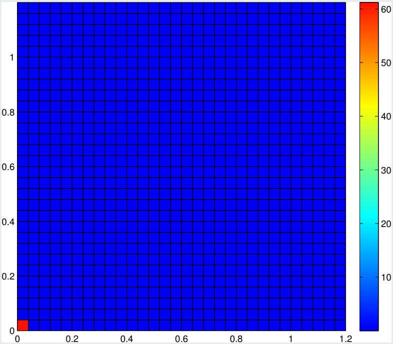
Updated or total Lagrangian formulation

- $\rho \dfrac{\mathrm{d}\,\mathrm{U}}{\mathrm{d}t} + \nabla_x \,\boldsymbol{.}\, \mathrm{F}(\mathrm{U}) = 0$ ou $\rho^0 \dfrac{\mathrm{d}\,\mathrm{U}}{\mathrm{d}t} + \nabla_X \,\boldsymbol{.}\, \left(|\mathrm{J}|\mathrm{J}^{-1}\mathrm{F}(\mathrm{U})\right) = 0$

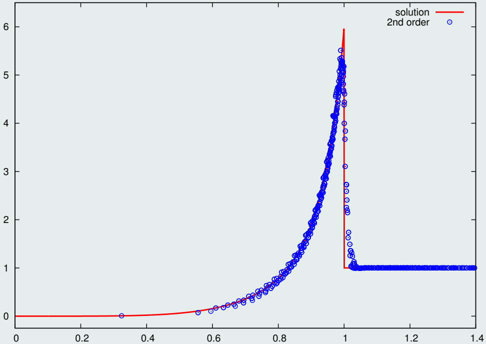
Piecewise polynomial approximation

- $\mathrm{U}_{h,c}^n(\boldsymbol{x})$ the polynomial approximation of the solution on ω_c^n
- $\mathrm{U}_{h,c}^n(\boldsymbol{X})$ the polynomial approximation of the solution on ω_c^0
- $\mathrm{U}_{qc} = \mathrm{U}_{h,c}^n(\boldsymbol{x}_q)$ (moving config.) or $\mathrm{U}_{qc} = \mathrm{U}_{h,c}^n(\boldsymbol{X}_q)$ (fixed config.)

Sedov point blast problem



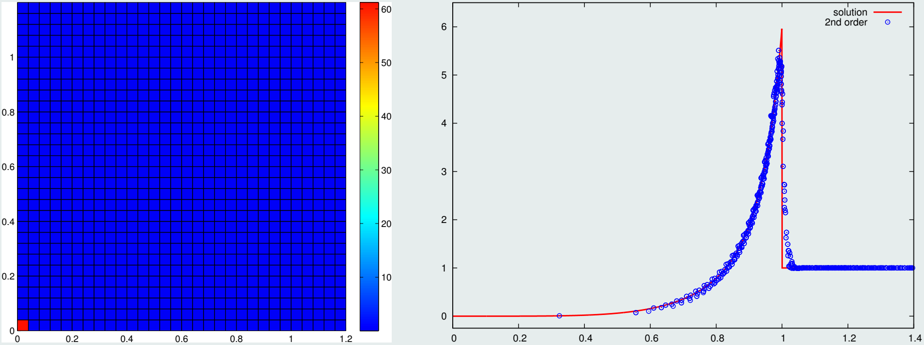
(a) Pressure field



(b) Density profiles

Figure : Solution at time $t = 1$ for a Sedov problem on a 30×30 Cartesian mesh

Sedov point blast problem



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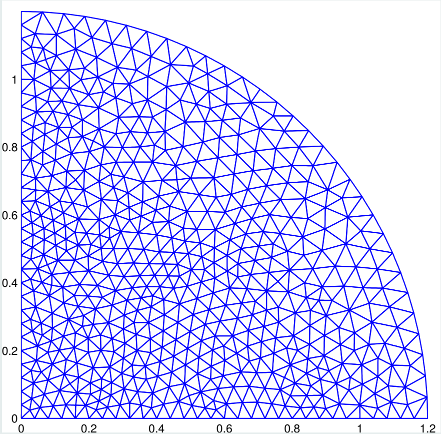
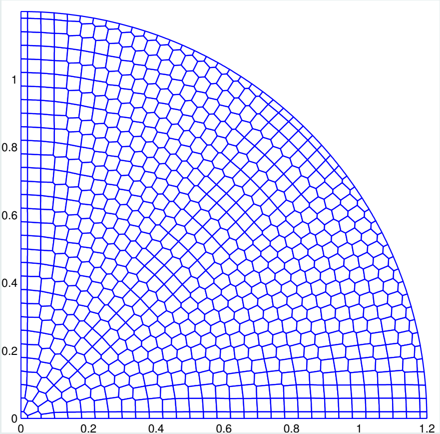


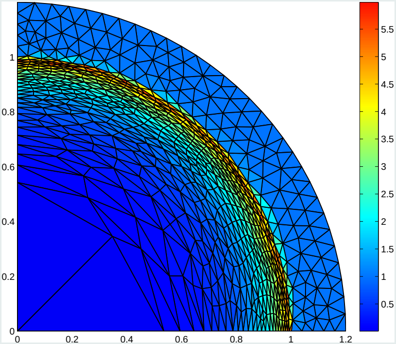
(c) Triangular grid - 1110 cells



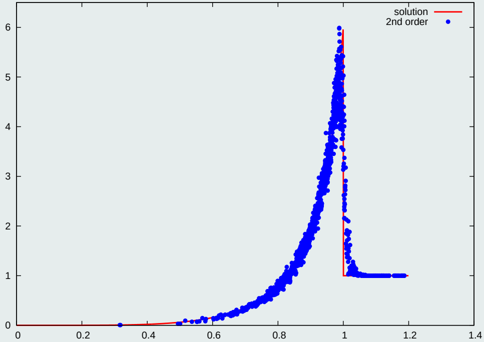
(d) Polygonal grid - 775 cells

Figure : Initial unstructured grids for Sedov point blast problem

Sedov point blast problem



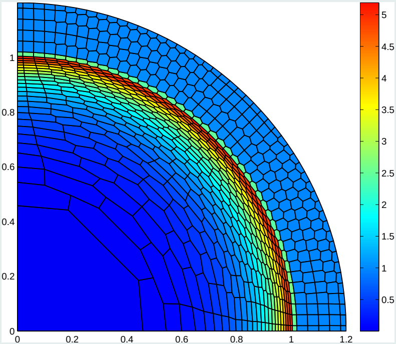
(e) Density field



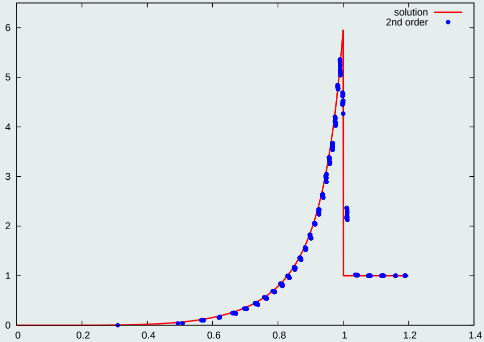
(f) Density profiles

Figure : Solution at time $t = 1$ for a Sedov problem on a grid made of 1110 triangular cells

Sedov point blast problem



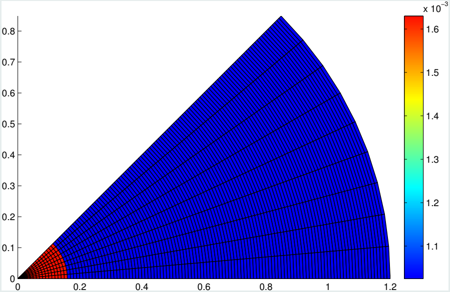
(g) Density field



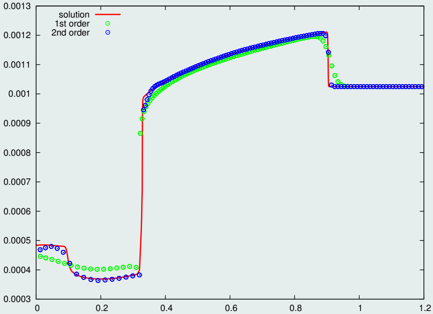
(h) Density profiles

Figure : Solution at time $t = 1$ for a Sedov problem on a grid made of 775 polygonal cells

Underwater TNT explosion



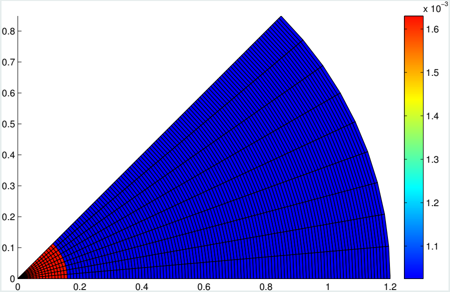
(i) Density field - 2nd order



(j) Density profiles

Figure : Solution at time $t = 2.5 \times 10^{-4}$ for a underwater TNT explosion on a 120×9 polar mesh

Underwater TNT explosion



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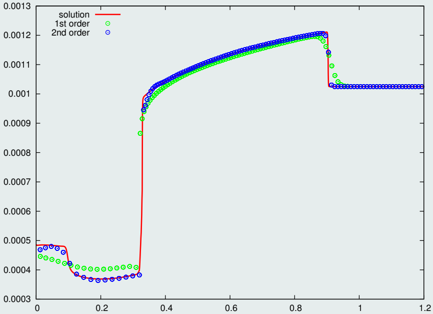
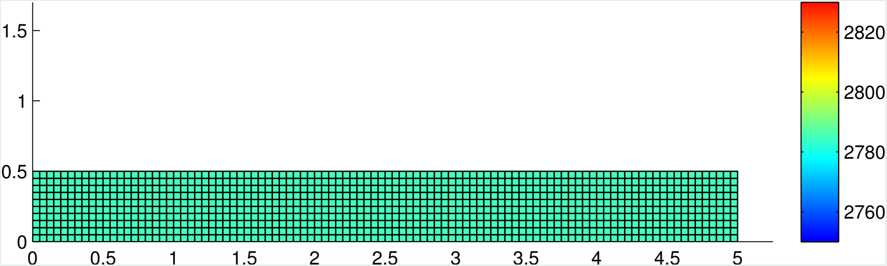


(j) Density profiles

Figure : Solution at time $t = 2.5 \times 10^{-4}$ for a underwater TNT explosion on a 120×9 polar mesh

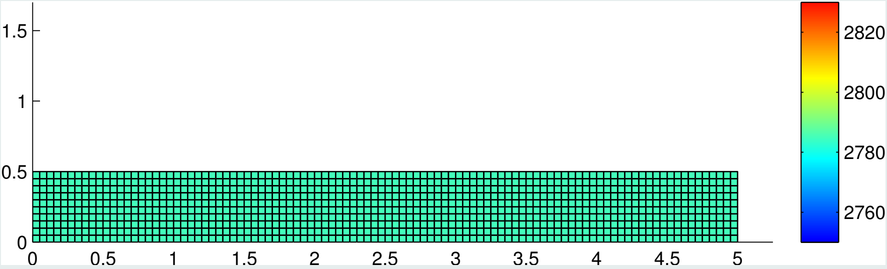
Aluminium projectile impact problem



(k) Density field

Figure : Solution at time $t = 0.05$ for a projectile impact problem on a 100×10 Cartesian mesh

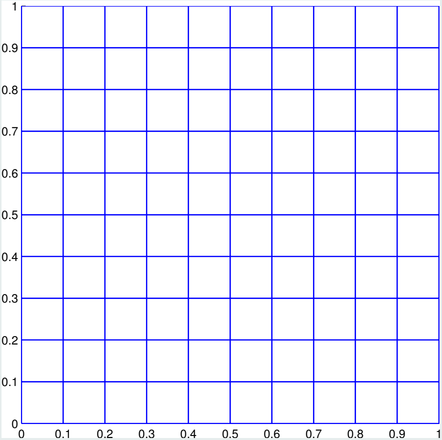
Aluminium projectile impact problem



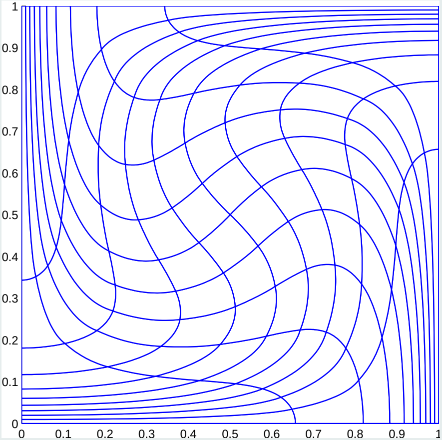
(k) Density field

Figure : Solution at time $t = 0.05$ for a projectile impact problem on a 100×10 Cartesian mesh

Taylor-Green vortex



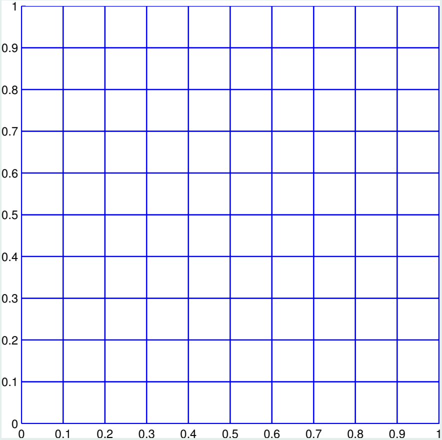
(l) 2nd order



(m) Exact solution

Figure : Final deformed grids at time $t = 0.75$, on a 10×10 Cartesian mesh

Taylor-Green vortex



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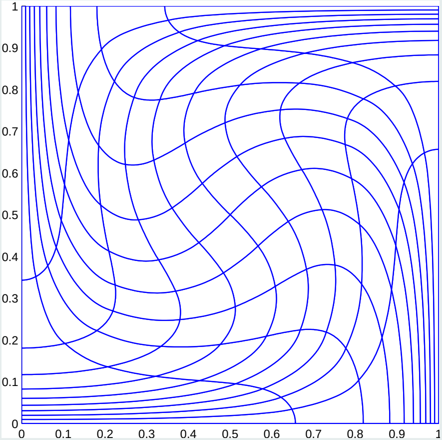

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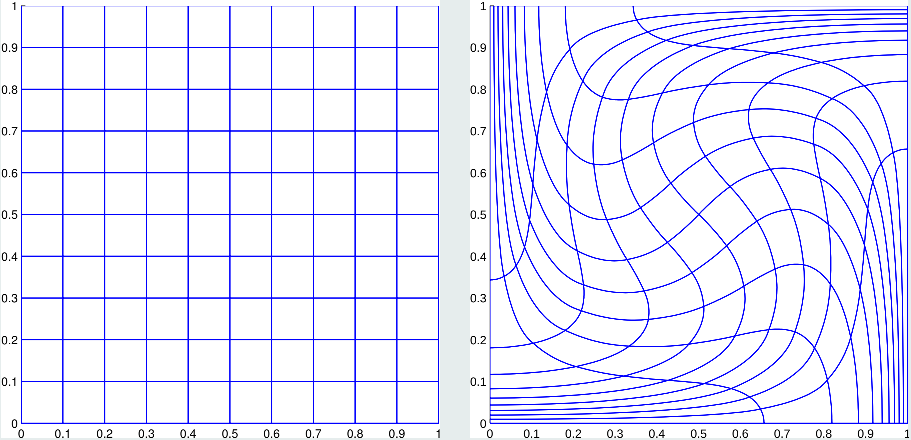
Figure : Final deformed grids at time $t = 0.75$, on a 10×10 Cartesian mesh

Convergence rates

	L_1		L_2		L_∞	
h	$E^h_{L_1}$	$q^h_{L_1}$	$E^h_{L_2}$	$q^h_{L_2}$	$E^h_{L_\infty}$	$q^h_{L_\infty}$
$\frac{1}{10}$	5.06E-3	1.94	6.16E-3	1.93	2.20E-2	1.84
$\frac{1}{20}$	1.32E-3	1.98	1.62E-3	1.97	5.91E-3	1.95
$\frac{1}{40}$	3.33E-4	1.99	4.12E-4	1.99	1.53E-3	1.98
$\frac{1}{80}$	8.35E-5	2.00	1.04E-4	2.00	3.86E-4	1.99
$\frac{1}{160}$	2.09E-5	-	2.60E-5	-	9.69E-5	-

Table: Convergence rates on the pressure for a 2nd order DG scheme

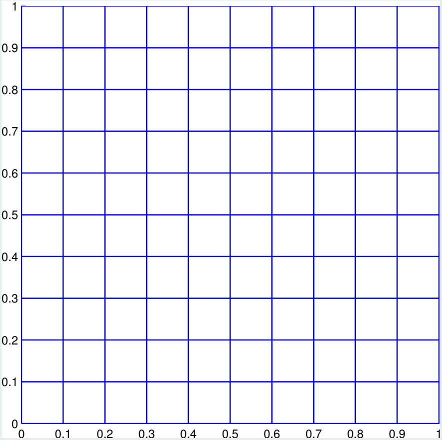
Taylor-Green vortex



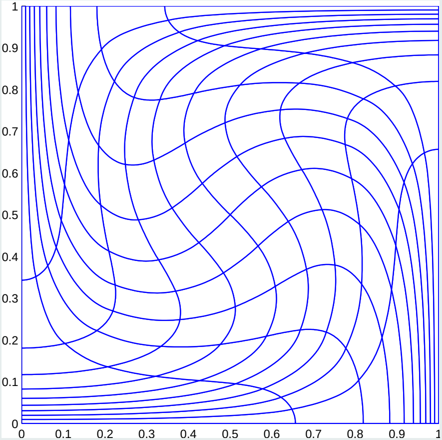
(n) 3rd order (o) Exact solution

Figure : Final deformed grids at time $t = 0.75$, on a 10×10 Cartesian mesh

Taylor-Green vortex



(n) 3rd order



(o) Exact solution

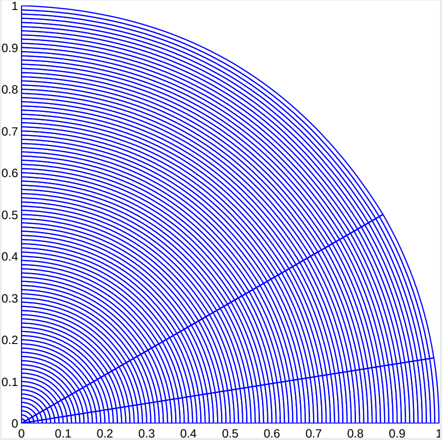
Figure : Final deformed grids at time $t = 0.75$, on a 10×10 Cartesian mesh

Convergence rates

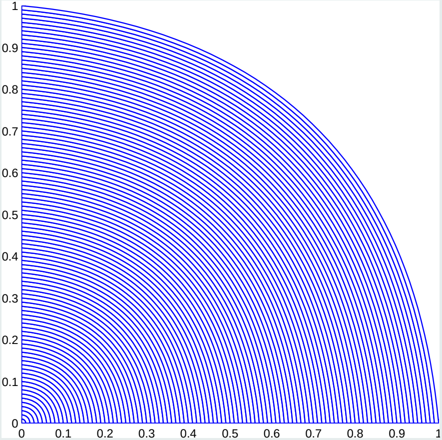
	L_1		L_2		L_∞	
h	$E^h_{L_1}$	$q^h_{L_1}$	$E^h_{L_2}$	$q^h_{L_2}$	$E^h_{L_\infty}$	$q^h_{L_\infty}$
$\frac{1}{10}$	2.67E-4	2.96	3.36E-7	2.94	1.21E-3	2.86
$\frac{1}{20}$	3.43E-5	2.97	4.36E-5	2.96	1.66E-4	2.93
$\frac{1}{40}$	4.37E-6	2.99	5.59E-6	2.98	2.18E-5	2.96
$\frac{1}{80}$	5.50E-7	2.99	7.06E-7	2.99	2.80E-6	2.99
$\frac{1}{160}$	6.91E-8	-	8.87E-8	-	3.53E-7	-

Table: Convergence rates on the pressure for a 3rd order DG scheme

Polar meshes - symmetry preservation



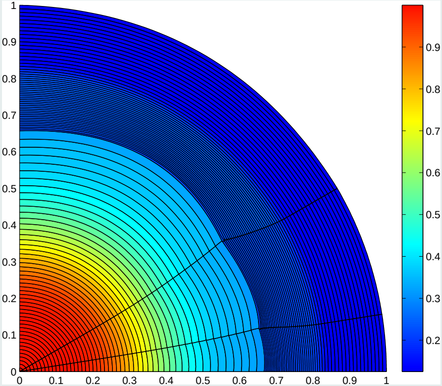
(p) 100 × 3



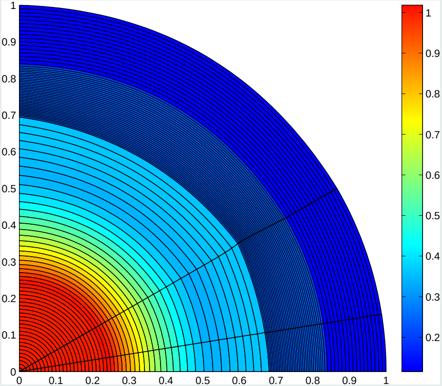
(q) 100 × 1

Figure : Curvilinear grids defined in polar coordinates

Sod shock tube problem - symmetry preservation



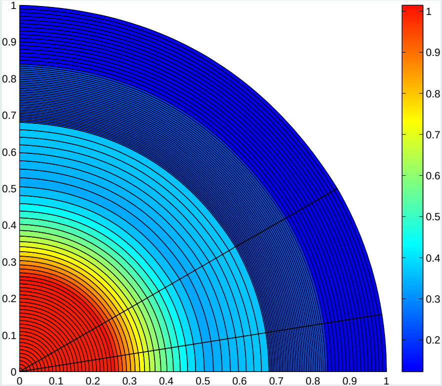
(r) 1st order



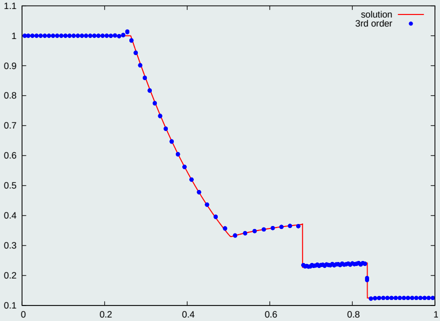
(s) 2nd order

Figure : Density fields with 1st and 2nd order schemes on a 3rd mesh

Sod shock tube problem - symmetry preservation



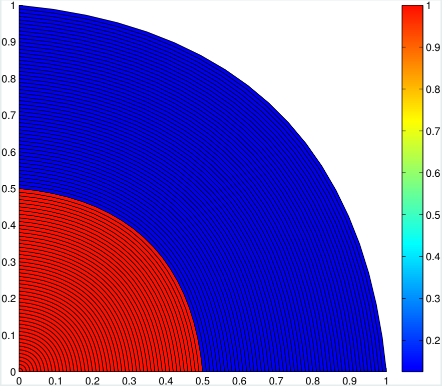
(t) Density field



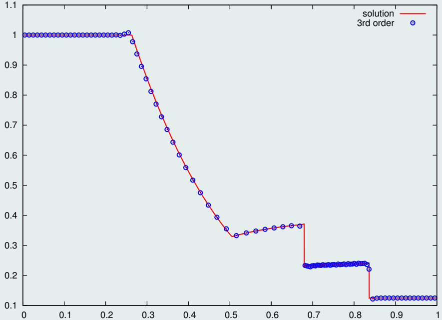
(u) Density profiles

Figure : 3rd order solution for a Sod shock tube problem on a 100×3 polar grid

Sod shock tube problem - symmetry preservation



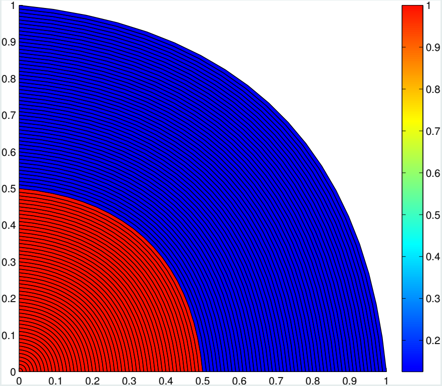
(v) Density field



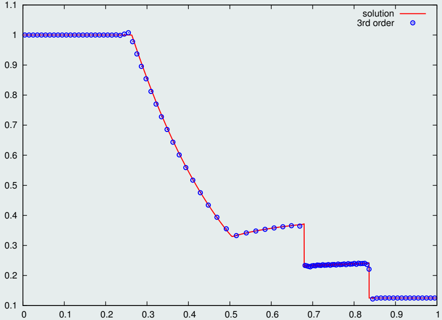
(w) Density profiles

Figure : 3rd order solution for a Sod shock tube problem on a 100×1 polar grid

Sod shock tube problem - symmetry preservation



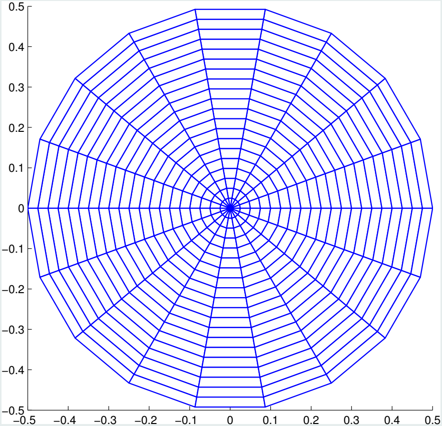
(v) Density field



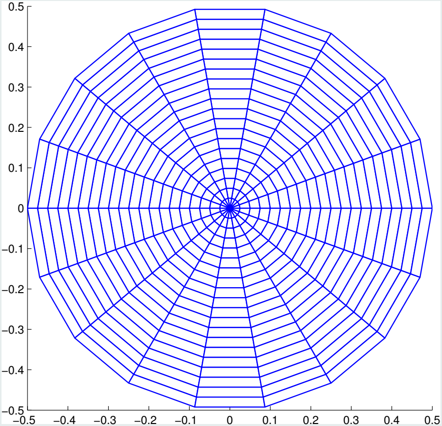
(w) Density profiles

Figure : 3rd order solution for a Sod shock tube problem on a 100×1 polar grid

Gresho-like vortex problem



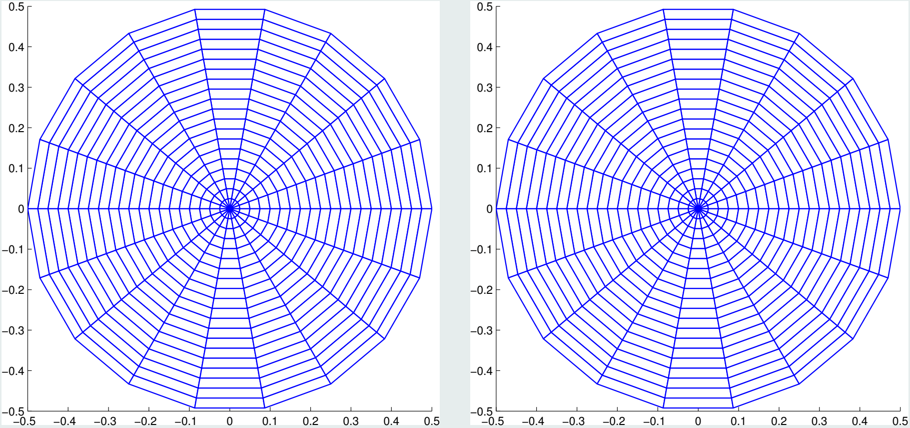
(a) 1st order



(b) 2nd order

Figure : Final deformed grids at time $t = 1$, on a 20×18 polar mesh

Gresho-like vortex problem

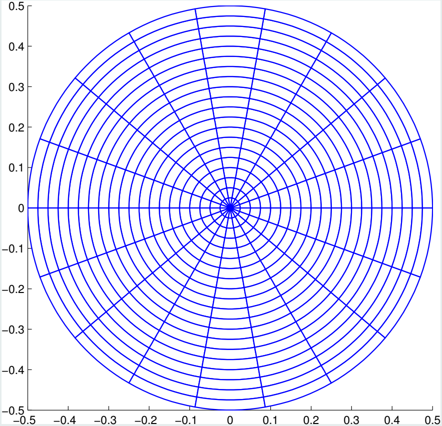


(a) 1st order

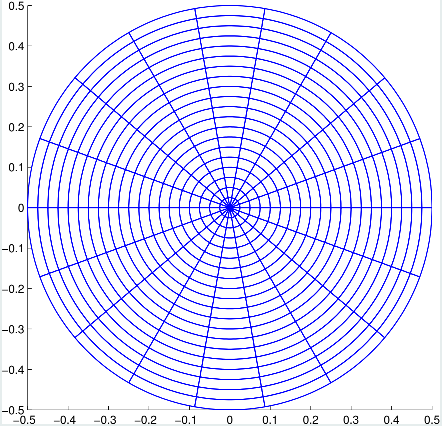
(b) 2nd order

Figure : Final deformed grids at time $t = 1$, on a 20×18 polar mesh

Gresho-like vortex problem



(c) 3rd order



(d) Exact solution

Figure : Final deformed grids at time $t = 1$, on a 20×18 polar mesh

Gresho-like vortex problem

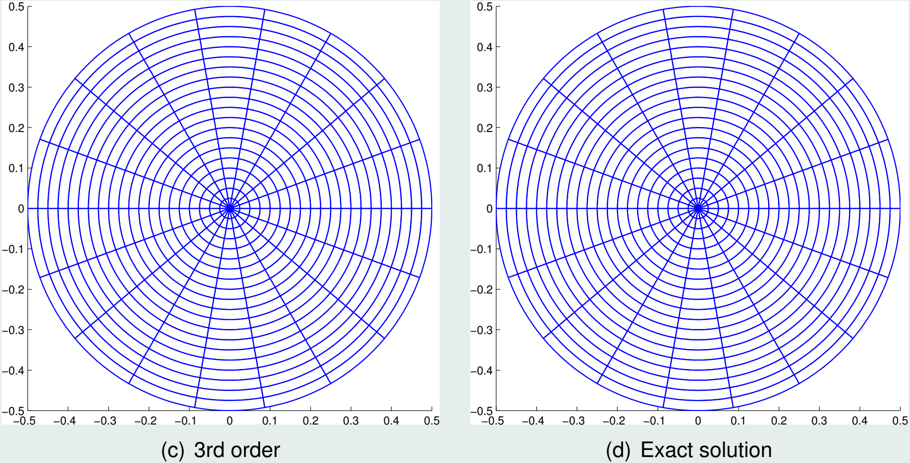
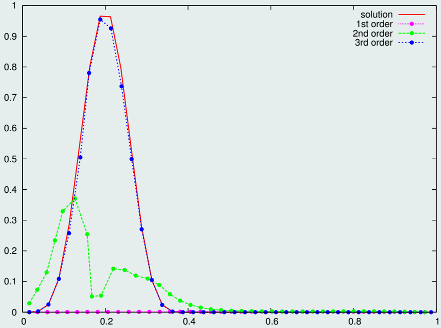
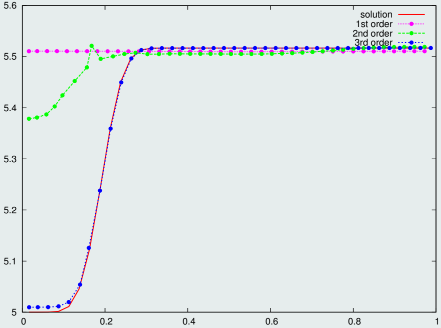


Figure : Final deformed grids at time $t = 1$, on a 20×18 polar mesh

Gresho-like vortex problem



(e) Velocity profiles



(f) Pressure profiles

Figure : Velocity and pressure profiles at time $t = 1$, on a 20×18 polar grid

Gresho-like vortex problem

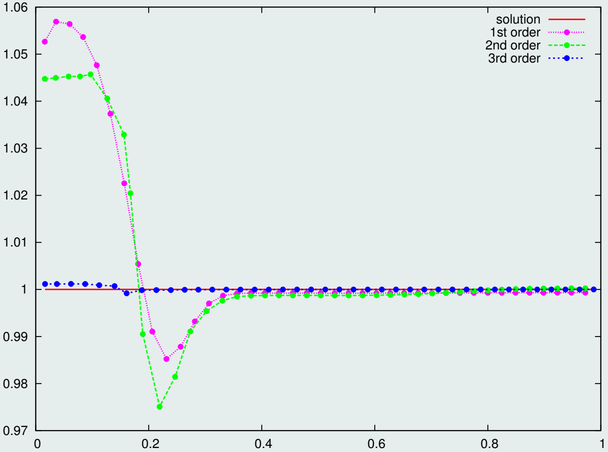
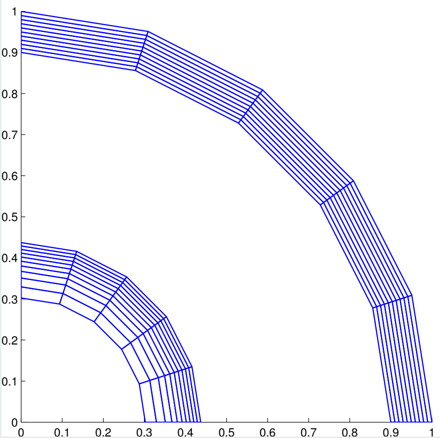
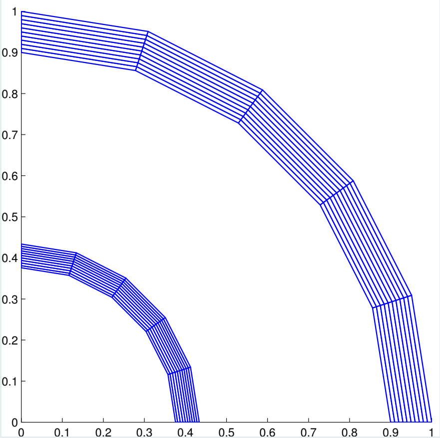


Figure : Density profiles at time $t = 1$, on a 20×18 polar grid

Kidder isentropic compression



(g) 1st order



(h) 2nd order

Figure : Intial and final grids for a Kidder problem on a 10×5 polar mesh

Kidder isentropic compression

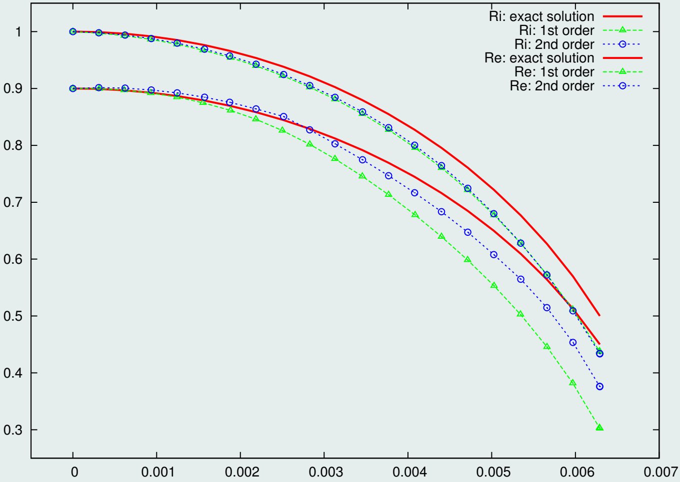
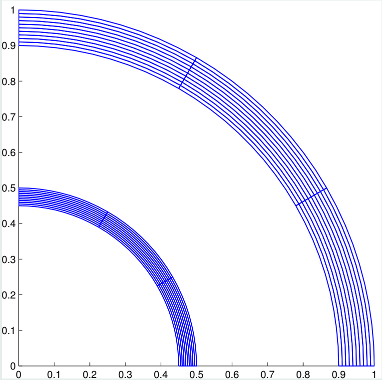
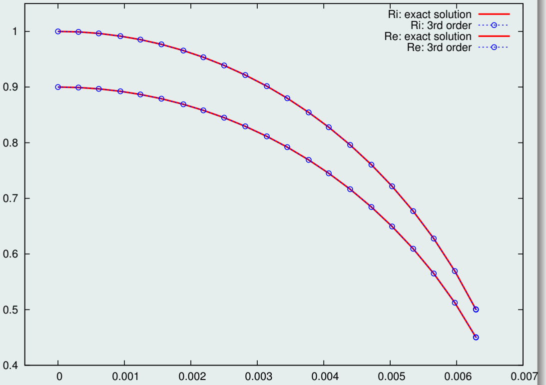


Figure : Interior and exterior shell radii evolution for a Kidder problem on a 10×5 polar mesh

Kidder isentropic compression



(i) Initial and final grids



(j) Shell radii evolution

Figure : 3rd order solution for a Kidder compression problem on a 10×3 polar grid

Accuracy and computational time for a Taylor-Green vortex

D.O.F	N	$E^h_{L_1}$	$E^h_{L_2}$	$E^h_{L_\infty}$	time (sec)
600	24×25	2.67E-2	3.31E-2	8.55E-2	2.01
2400	48×50	1.36E-2	1.69E-2	4.37E-2	11.0

Table: 1st order scheme

D.O.F	N	$E^h_{L_1}$	$E^h_{L_2}$	$E^h_{L_\infty}$	time (sec)
630	14×15	2.76E-3	3.33E-3	1.07E-2	2.77
2436	28×29	7.52E-4	9.02E-4	2.73E-3	11.3

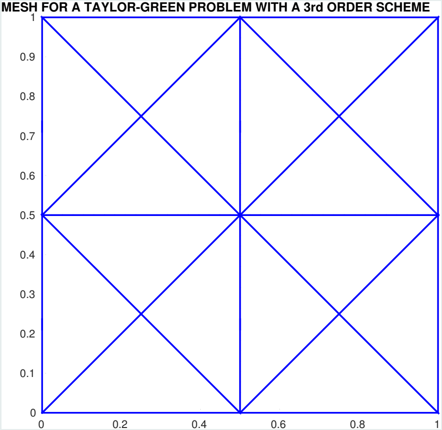
Table: 2nd order scheme

D.O.F	N	$E^h_{L_1}$	$E^h_{L_2}$	$E^h_{L_\infty}$	time (sec)
600	10×10	2.67E-4	3.36E-4	1.21E-3	4.00
2400	20×20	3.43E-5	4.36E-5	1.66E-4	30.6

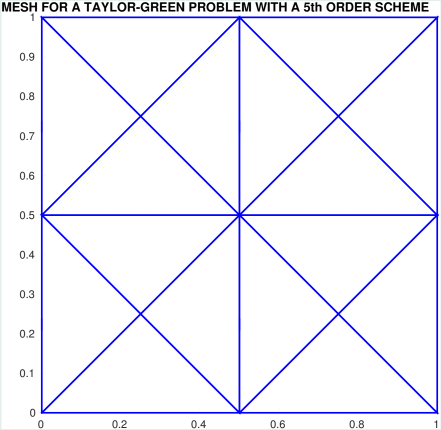
Table: 3rd order scheme

- F. VILAR, P.-H. MAIRE AND R. ABGRALL, *Cell-centered discontinuous Galerkin discretizations for two-dimensional scalar conservation laws on unstructured grids and for one-dimensional Lagrangian hydrodynamics.* CAF, 2010.
- F. VILAR, *Cell-centered discontinuous Galerkin discretization for two-dimensional Lagrangian hydrodynamics.* CAF, 2012.
- F. VILAR, P.-H. MAIRE AND R. ABGRALL, *A discontinuous Galerkin discretization for solving the two-dimensional gas dynamics equations written under total lagrangian formulation on general unstructured grids.* JCP, 2014.
- F. VILAR, C.-W. SHU AND P.-H. MAIRE, *Positivity-preserving cell-centered Lagrangian schemes for multi-material compressible flows: Form first-order to high-orders. Part I: The 1D case.* JCP, 2016.
- F. VILAR, C.-W. SHU AND P.-H. MAIRE, *Positivity-preserving cell-centered Lagrangian schemes for multi-material compressible flows: Form first-order to high-orders. Part II: The 2D case.* JCP, 2016.

Taylor-Green vortex



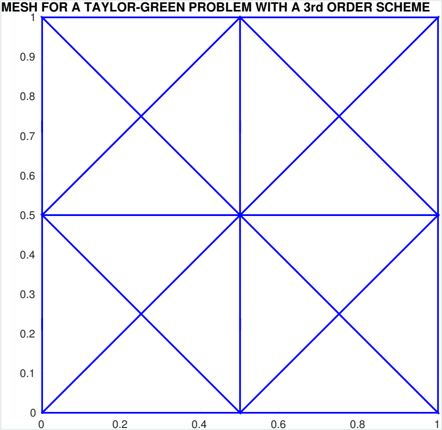
(k) 3rd order



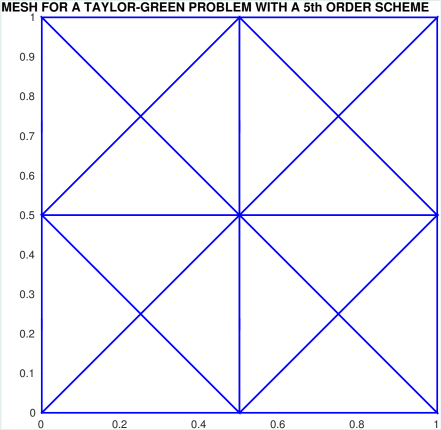
(l) 5th order

Figure : Final deformed grids at time $t = 0.6$, for 16 triangular cells meshes

Taylor-Green vortex



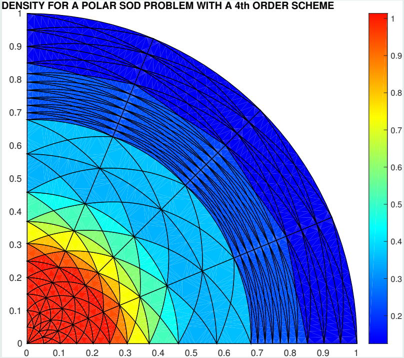
(k) 3rd order



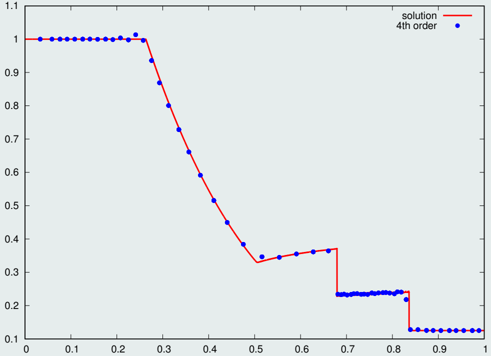
(l) 5th order

Figure : Final deformed grids at time $t = 0.6$, for 16 triangular cells meshes

Sod shock tube problem - symmetry preservation



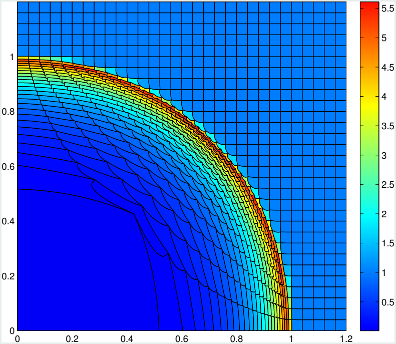
(m) Density field



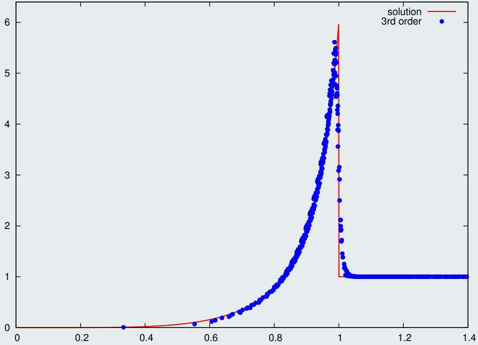
(n) Density profiles

Figure : 4th order solution for a Sod shock tube problem on a polar grid made of 308 triangular cells

Sedov point blast problem - spurious deformations



(o) Density field



(p) Density profiles

Figure : Third-order solution at time $t = 1$ for a Sedov problem on a 30×30 Cartesian mesh