Cell-centered discontinuous Galerkin scheme for Lagrangian hydrodynamics

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energie atomique • energies alternatives

1. Introduction
   - Discontinuous Galerkin (DG)
   - Scalar conservation laws
   - 1D Lagrangian hydrodynamics
2. 2D Lagrangian hydrodynamics
   - References
   - System and equations
   - Geometric consideration
   - 2nd order Deformation tensor
   - 2nd order DG scheme
3. Conclusion

- extension of finite volumes method
- polynomial approximation of the solution in the cells
- high order scheme, high precision

- local variational formulation
- choice of the numerical fluxes (global L^2 stability, entropic inequality)
- time discretization - TVD multistep Runge-Kutta

C.-W. Shu, *Discontinuous Galerkin methods: General approach and stability*, 2008

- limitation - vertex-based hierarchical slope limiters

D. Kuzmin, *A vertex-based hierarchical slope limiter for p-adaptive discontinuous Galerkin methods* J. Comp. Appl. Math., 2009

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comparison between the second order and the third order scheme with limitation

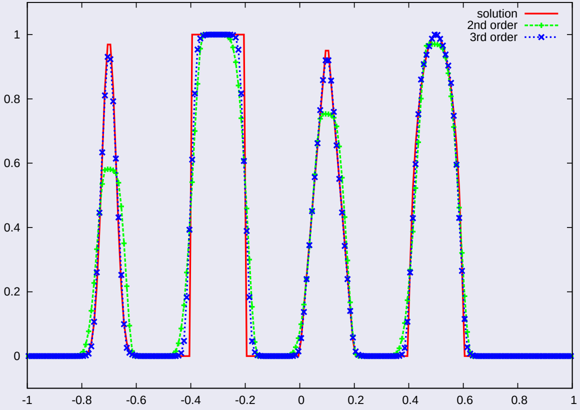
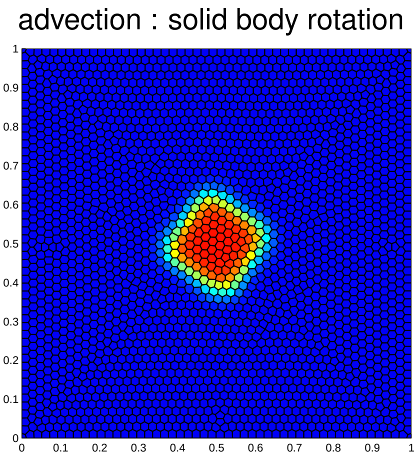
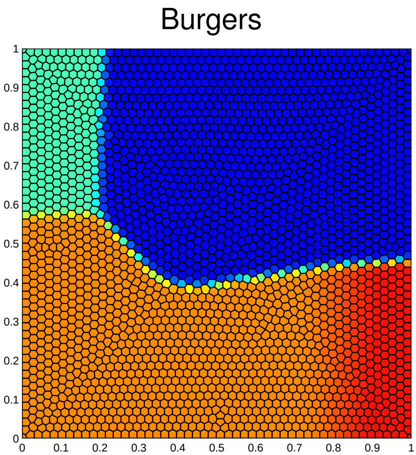


Figure: linear advection of a combination of smooth and discontinuous profiles





numerical solutions using third order limited DG on a polygonal grid made of 2500 cells

rate of convergence with and without the slope limitation

		L_1	L_2
linear advection	first order	1.02	1.02
	second order	1.99	1.98
	second order lim	2.15	2.15
	third order	2.98	2.98
	third order lim	3.45	3.22

Table: for the smooth solution $u_0(\boldsymbol{x}) = \sin(2\pi x)\sin(2\pi y)$ on a $[0,1]^2$ Cartesian grid

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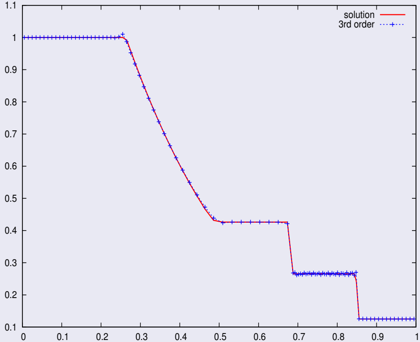
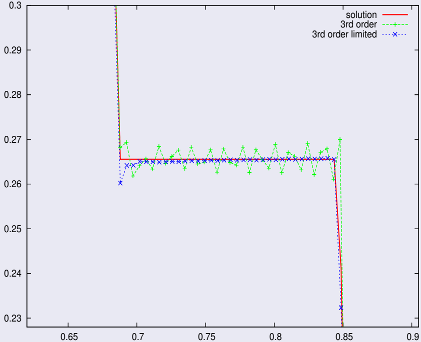
influence of the limitation on the linearized Riemann invariants



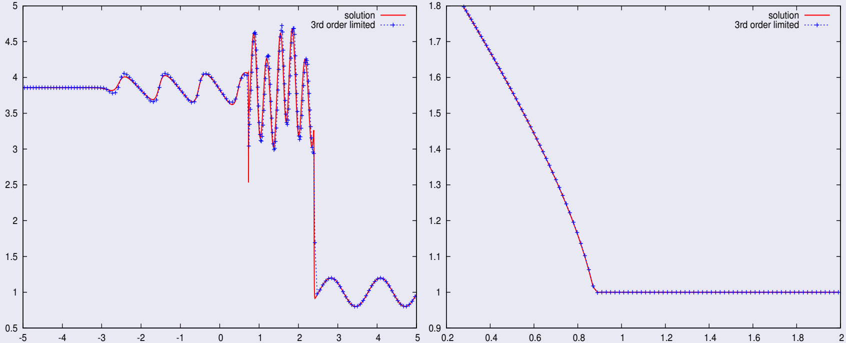
(a) without limitation



(b) with and without limitation

Figure: third order DG for the Sod shock tube problem using 100 cells: density

3rd order DG scheme with limitation: density



(a) Shu oscillating shock tube (b) uniformly accelerated piston

P.-H. MAIRE, *A high-order cell-centered Lagrangian scheme for two-dimensional compressible fluid flows on unstructured meshes* J. Comp. Phys., 2009

rate of convergence with and without the slope limitation

		L_1	L_2
gas dynamics	first order	0.80	0.73
	second order	2.25	2.26
	second order lim	2.04	2.21
	third order	3.39	3.15
	third order lim	2.75	2.72

Table: for a smooth solution in the special case $\gamma = 3$

F. Vilar, P.-H. Maire, R. Abgrall, *Cell-centered discontinuous Galerkin discretizations for two-dimensional scalar conservation laws on unstructured grids and for one-dimensional Lagrangian hydrodynamics*, Comp. & Fluids, 2010

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F. L. Adessio, D. E. Carroll, J. K. Dukowicz, N. L. Johnson, B. A. Kashiwa, M. E. Maltrud, H. M. Ruppel, *Caveat: a computer code for fluid dynamics problems with large distortion and internal slip*, Los Alamos National Laboratory, 1986

R. Loubère, *Une Méthode Particulaire Lagrangienne de type Galerkin Discontinu. Application à la Mécanique des Fluides et l'Interaction Laser/Plasma*, PhD thesis, Université Bordeaux I, 2002

B. Després, C. Mazeran, *Lagrangian Gas Dynamics in Two Dimensions and Lagrangian systems*, Arch. Rational Mech. Anal., 2005

P.-H. Maire, R. Abgrall, J. Breil, J. Ovadia, *A cell-centered Lagrangian scheme for two-dimensional compressible flow problems*, SIAM J. Sci. Comp, 2007

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- gas dynamics system in Lagrangian formalism

$$\rho^0 \frac{d}{dt}(\frac{1}{\rho}) - \nabla_{\boldsymbol{X}} \cdot (J\mathrm{F}^{-1}\boldsymbol{U}) = 0 \tag{1a}$$

$$\rho^0 \frac{d\boldsymbol{U}}{dt} + \nabla_{\boldsymbol{X}} \cdot (J\mathrm{F}^{-t}P) = \boldsymbol{0} \tag{1b}$$

$$\rho^0 \frac{dE}{dt} + \nabla_{\boldsymbol{X}} \cdot (J\mathrm{F}^{-1}P\boldsymbol{U}) = 0 \tag{1c}$$

  where $\boldsymbol{X}$ is the Lagrangian (initial) coordinate

- $\mathrm{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}$ is called the deformation gradient tensor, where $\boldsymbol{x}$ is the Eulerian (actual) coordinate and $J = \det(\mathrm{F})$

- using the trajectory equation $\frac{d\boldsymbol{x}}{dt} = \boldsymbol{U}(\boldsymbol{x}, t) \Longrightarrow \frac{d\mathrm{F}}{dt} = \nabla_{\boldsymbol{X}}\boldsymbol{U}$ (2)

- Piola compatibility condition $\nabla_{\boldsymbol{X}} \cdot (J\mathrm{F}^{-t}) = \boldsymbol{0}$ (3)

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- being given a mapping $\boldsymbol{x} = \boldsymbol{\Phi}(\boldsymbol{X}, t)$

$$\mathsf{F} = \nabla_{\boldsymbol{X}} \boldsymbol{\Phi} \tag{4}$$

- developing $\boldsymbol{\Phi}$ on the basis functions λ_p in the cell Ω_c

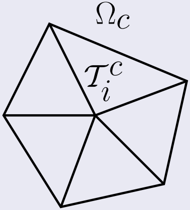
$$\begin{aligned} \boldsymbol{\Phi}_h^c(\boldsymbol{X}, t) &= \boldsymbol{\Phi}_h(\boldsymbol{X}, t)_{|\Omega_c} \\ &= \sum_p \lambda_p(\boldsymbol{X}) \, \boldsymbol{\Phi}_p(t) \end{aligned}$$

where the p points are some control points

- by setting $\mathsf{G}_c = (J\mathsf{F}^{-t})_c$

$$\nabla_{\boldsymbol{X}} \cdot \mathsf{G}_c = \sum_p \begin{pmatrix} \Phi_p^Y(\partial_{YX}\lambda_p - \partial_{XY}\lambda_p) \\ -\Phi_p^X(\partial_{YX}\lambda_p - \partial_{XY}\lambda_p) \end{pmatrix} = \mathbf{0}$$

- using (4) and $\frac{d}{dt}\mathbf{\Phi}_p = \mathbf{U}_p \quad \Longrightarrow \quad \frac{d}{dt}\mathrm{F}_c = \sum_p \mathbf{U}_p \otimes \nabla_{\mathbf{X}} \lambda_p$ (5)
- in 2D, $\mathrm{F} \longrightarrow J\mathrm{F}^{-t} = \mathrm{G}$ is a linear function
- $J\mathrm{F}^{-t}\mathbf{N}$ represents the geometric normal in the Eulerian frame thanks to Nanson formula $J\mathrm{F}^{-t}\mathbf{N}\mathrm{d}S = \mathrm{G}\mathbf{N}\mathrm{d}S = \mathbf{n}\mathrm{d}s$
- to ensure this quantity to be continuous, we discretize F by means of mapping defined on triangular cells $\mathcal{T}_i^c$ with $i = 1 \dots ntri$, using finite elements polynomial basis



- using the fact $\frac{d}{dt}\mathrm{F} = \nabla_{\mathbf{X}}\mathbf{U}$, F approximation order has to be one less than the one obtain with the DG scheme on $\frac{1}{\rho}$, $\mathbf{U}$ and E

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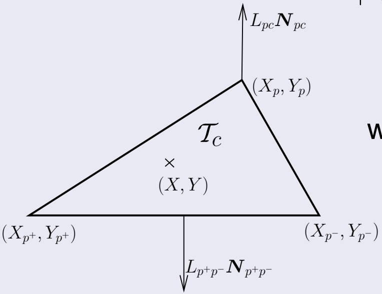
- for the P_1 representation, the chosen finite elements polynomial basis in a general triangle $\mathcal{T}_c$ write

$$\lambda_p(\boldsymbol{X}) = \frac{1}{2|\mathcal{T}_c|}[X(Y_{p^+} - Y_{p^-}) - Y(X_{p^+} - X_{p^-}) + X_{p^+}Y_{p^-} - X_{p^-}Y_{p^+}] \quad (6)$$

- we can access to $\nabla_{\boldsymbol{X}}\lambda_p$ needed in (5)

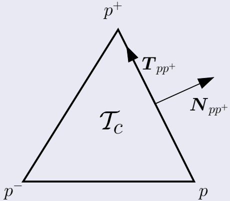
$$\nabla_{\boldsymbol{X}}\lambda_p(\boldsymbol{X}) = \frac{1}{2|\mathcal{T}_c|}\begin{pmatrix} Y_{p^+} - Y_{p^-} \\ X_{p^-} - X_{p^+} \end{pmatrix} = \frac{1}{|\mathcal{T}_c|}L_{pc}\boldsymbol{N}_{pc} \quad (7)$$



where $$L_{pc}\boldsymbol{N}_{pc} = \frac{L_{p^-p}\boldsymbol{N}_{p^-p} + L_{pp^+}\boldsymbol{N}_{pp^+}}{2} = -\frac{L_{p^+p^-}\boldsymbol{N}_{p^+p^-}}{2}$$

- the equation (5) rewrites $$\frac{d}{dt}\mathsf{F}_c = \frac{1}{|\mathcal{T}_c|} \sum_{p \in \mathcal{P}(\mathcal{T}_c)} \boldsymbol{U}_p \otimes L_{pc} \boldsymbol{N}_{pc} \qquad (8)$$

- with this definition, G$\boldsymbol{N}$ continuity is well preserved at the interface betweens triangles



$$\mathsf{G}_c\, L_{pp^+} \boldsymbol{N}_{pp^+} = \frac{1}{|\mathcal{T}_c|} \sum_{p_t \in \mathcal{P}(\mathcal{T}_c)} L_{p_t c}\, L_{pp^+} \begin{pmatrix} \Phi_p^Y (N^X_{pp^+} N^Y_{p_t c} - N^Y_{pp^+} N^X_{p_t c}) \\ -\Phi_p^X (N^X_{pp^+} N^Y_{p_t c} - N^Y_{pp^+} N^X_{p_t c}) \end{pmatrix}$$

$$= \frac{1}{|\mathcal{T}_c|} \sum_{p_t \in \mathcal{P}(\mathcal{T}_c)} (L_{pp^+} \boldsymbol{T}_{pp^+} \cdot L_{p_t c} \boldsymbol{N}_{p_t c}) \begin{pmatrix} \Phi_p^Y \\ -\Phi_p^X \end{pmatrix}$$

$$= \begin{pmatrix} \Phi^Y_{p^+} - \Phi^Y_p \\ \Phi^X_p - \Phi^X_{p^+} \end{pmatrix} = \begin{pmatrix} y_{p^+} - y_p \\ x_p - x_{p^+} \end{pmatrix} = l_{pp^+} \boldsymbol{n}_{pp^+} \qquad (9)$$

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Discontinuous Galerkin

- $\{\sigma_k^c\}_{k=0\dots K}$ basis of $\mathbb{P}^{order-1}(\Omega_c)$
- $\phi_h^c(\boldsymbol{X},t) = \sum_{k=0}^{K} \phi_k^c(t)\sigma_k^c(\boldsymbol{X})$ approximate of $\phi(\boldsymbol{X},t)$ on Ω_c
- Taylor basis, $k_1 + k_2 = k$

$$\sigma_k^c = \frac{1}{k_1!k_2!}[(\frac{X-X_c}{\Delta X_c})^{k_1}(\frac{Y-Y_c}{\Delta Y_c})^{k_2} - \langle(\frac{X-X_c}{\Delta X_c})^{k_1}(\frac{Y-Y_c}{\Delta Y_c})^{k_2}\rangle]$$

- for the second order scheme, $K = 2$

$$\sigma_0^c = 1,\ \sigma_1^c = \frac{X-X_c}{\Delta X_c},\ \sigma_2^c = \frac{Y-Y_c}{\Delta Y_c}$$

where $\Delta X_c = \frac{X_{max}-X_{min}}{2}$ and $\Delta Y_c = \frac{Y_{max}-Y_{min}}{2}$ with X_{max}, Y_{max}, X_{min}, Y_{min} the maximum and minimum coordinates in the cell Ω_c

Density

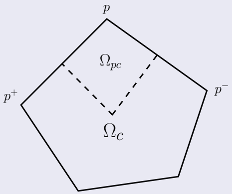
- local variational formulation of (1a) on Ω_c

$$\int_{\Omega_c} \rho^0 \frac{d}{dt}(\frac{1}{\rho})\sigma_q \mathrm{d}\Omega = \sum_{k=0}^{K} \frac{d}{dt}(\frac{1}{\rho})_k \int_{\Omega_c} \rho^0 \sigma_q \sigma_k \mathrm{d}\Omega$$
$$= \int_{\Omega_c} \sigma_q \nabla_{\boldsymbol{X}} \cdot (\mathsf{J}\mathsf{F}^{-1}\boldsymbol{U})\mathrm{d}\Omega$$
$$= -\int_{\Omega_c} \boldsymbol{U} \cdot \mathsf{J}\mathsf{F}^{-t} \nabla_{\boldsymbol{X}} \sigma_q \mathrm{d}\Omega + \int_{\partial\Omega_c} \overline{\boldsymbol{U}} \cdot \sigma_q \mathsf{J}\mathsf{F}^{-t} \boldsymbol{N} \mathrm{d}L$$

- $\mathsf{G}_i^c = (\mathsf{J}\mathsf{F}^{-t})_i^c$ is constant on $\mathcal{T}_i^c$ and $\nabla_{\boldsymbol{X}}\sigma_q$ over Ω_c

$$\int_{\Omega_c} \rho^0 \frac{d}{dt}(\frac{1}{\rho})\sigma_q \mathrm{d}\Omega = -\sum_{i=1}^{ntri} \mathsf{G}_i^c \nabla_{\boldsymbol{X}} \sigma_q \cdot \int_{\mathcal{T}_i^c} \boldsymbol{U} \mathrm{d}\mathcal{T} + \int_{\partial\Omega_c} \overline{\boldsymbol{U}} \cdot \sigma_q \mathsf{G} \boldsymbol{N} \mathrm{d}L$$

$$\int_{\Omega_c} \rho^0 \frac{d}{dt}\left(\frac{1}{\rho}\right)\sigma_q \mathrm{d}\Omega \simeq -\sum_{i=1}^{ntri} \mathrm{G}_i^c \nabla_{\boldsymbol{X}} \sigma_q \cdot \int_{\mathcal{T}_i^c} \boldsymbol{U} \mathrm{d}\mathcal{T} + \sum_{p \in \mathcal{P}(\Omega_c)} \boldsymbol{U}_p \cdot \underbrace{\int_{\partial\Omega_c \cap \partial\Omega_{pc}} \sigma_q \mathrm{G} \boldsymbol{N} \mathrm{d}L}_{l_{pc}^q \boldsymbol{n}_{pc}^q}$$



- finally, the equation on the density leads to

$$\int_{\Omega_c} \rho^0 \frac{d}{dt}\left(\frac{1}{\rho}\right)\sigma_q \mathrm{d}\Omega = -\sum_{i=1}^{ntri} \mathrm{G}_i^c \nabla_{\boldsymbol{X}} \sigma_q \cdot \int_{\mathcal{T}_i^c} \boldsymbol{U} \mathrm{d}\mathcal{T} + \sum_{p \in \mathcal{P}(\Omega_c)} \boldsymbol{U}_p \cdot l_{pc}^q \boldsymbol{n}_{pc}^q \qquad (10)$$

- for the first order with $l_{pc}\boldsymbol{n}_{pc} = l_{pc}^0 \boldsymbol{n}_{pc}^0$

$$m_c \frac{d}{dt}\left(\frac{1}{\rho}\right)_c = \int_{\Omega_c} \rho^0 \frac{d}{dt}\left(\frac{1}{\rho}\right) \mathrm{d}\Omega = \sum_{p \in \mathcal{P}(\Omega_c)} \boldsymbol{U}_p \cdot l_{pc} \boldsymbol{n}_{pc} \qquad (11)$$

Velocity

- local variational formulation of (1b) on Ω_c leads to

$$\int_{\Omega_c} \rho^0 \frac{d\,\boldsymbol{U}}{dt} \sigma_q \mathrm{d}\Omega = \sum_{i=1}^{ntri} \mathrm{G}_i^c \nabla_{\boldsymbol{X}} \sigma_q \int_{\mathcal{T}_i^c} P \mathrm{d}\mathcal{T} - \sum_{p \in \mathcal{P}(\Omega_c)} \boldsymbol{F}_{pc}^q \tag{12}$$

where $\boldsymbol{F}_{pc}^q = \int_{\partial\Omega_c \cap \partial\Omega_{pc}} \overline{P}\, \sigma_q \mathrm{G} \boldsymbol{N} \mathrm{d}L$

- for the first order with $\boldsymbol{F}_{pc} = \boldsymbol{F}_{pc}^0$

$$m_c \frac{d\,\boldsymbol{U}_c}{dt} = \int_{\Omega_c} \rho^0 \frac{d\,\boldsymbol{U}}{dt} \mathrm{d}\Omega = - \sum_{p \in \mathcal{P}(\Omega_c)} \boldsymbol{F}_{pc} \tag{13}$$

Energy

- local variational formulation of (1c) on Ω_c

$$\int_{\Omega_c} \rho^0 \frac{d\,E}{dt} \sigma_q \mathrm{d}\Omega = \sum_{i=1}^{ntri} \mathrm{G}_i^c \nabla_{\boldsymbol{X}} \sigma_q \cdot \int_{\mathcal{T}_i^c} \overline{P\,\boldsymbol{U}} \mathrm{d}\mathcal{T} - \sum_{p \in \mathcal{P}(\Omega_c)} \int_{\partial\Omega_c \cap \partial\Omega_{pc}} \overline{P\,\boldsymbol{U}} \cdot \sigma_q \mathrm{G}\boldsymbol{N} \mathrm{d}L \quad (14)$$

- we make the following fundamental assumption $\overline{P\,\boldsymbol{U}} = \overline{P}\,\overline{\boldsymbol{U}}$
- finally, the equation on the energy rewrites

$$\int_{\Omega_c} \rho^0 \frac{d\,E}{dt} \sigma_q \mathrm{d}\Omega = \sum_{i=1}^{ntri} \mathrm{G}_i^c \nabla_{\boldsymbol{X}} \sigma_q \cdot \int_{\mathcal{T}_i^c} P\,\boldsymbol{U} \mathrm{d}\mathcal{T} - \sum_{p \in \mathcal{P}(\Omega_c)} \boldsymbol{U}_p \cdot \boldsymbol{F}_{pc}^q \quad (15)$$

- for the first order

$$m_c \frac{d\,E_c}{dt} = \int_{\Omega_c} \rho^0 \frac{d\,E}{dt} \mathrm{d}\Omega = -\sum_{p \in \mathcal{P}(\Omega_c)} \boldsymbol{U}_p \cdot \boldsymbol{F}_{pc} \quad (16)$$

Entropic analysis

- the use of variational formulations and Gibbs formula leads to

$$\int_{\Omega_c} \rho^0\, T \frac{d\,S}{dt} \mathrm{d}\Omega = \int_{\partial\Omega_c} [\overline{P}\,\boldsymbol{U} + P\,\overline{\boldsymbol{U}} - \overline{P\,\boldsymbol{U}} - P\,\boldsymbol{U}]\,.\,\mathrm{G}\boldsymbol{N}\mathrm{d}L$$
$$= \sum_{f\in\mathcal{F}(\Omega_c)} \int_f (\overline{P} - P)(\boldsymbol{U} - \overline{\boldsymbol{U}})\,.\,\mathrm{G}\boldsymbol{N}\mathrm{d}L \qquad (17)$$

- a sufficient condition to satisfy $\int_{\Omega_c} \rho^0\, T \frac{d\,S}{dt} \mathrm{d}\Omega \geq 0$ consists in setting

$$\overline{P}(\boldsymbol{X_f}) = Pc(\boldsymbol{X_f}) - Z_c\,(\overline{\boldsymbol{U}}(\boldsymbol{X_f}) - \boldsymbol{U}_c(\boldsymbol{X_f}))\,.\,\frac{\mathrm{G}\boldsymbol{N}}{\|\mathrm{G}\boldsymbol{N}\|} \qquad (18)$$

where $\boldsymbol{X}_f$ is a point on the face f and Z_c a positive constant with a physical dimension of a density times a velocity

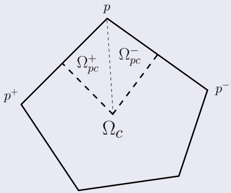
- using this expression to calculate $\boldsymbol{F}_{pc}^{q}$ leads to

$$
\begin{aligned}
\boldsymbol{F}_{pc}^{q} &= \int_{\partial\Omega_c\cap\partial\Omega_{pc}} \overline{P}\;\sigma_q J\mathsf{F}^{-t}\boldsymbol{N}\mathrm{d}L \\
&= \int_{\partial\Omega_c\cap\partial\Omega_{pc}} P_c\;\sigma_q \mathsf{G}\boldsymbol{N}\mathrm{d}L - \int_{\partial\Omega_c\cap\partial\Omega_{pc}} Z_c\,(\overline{\boldsymbol{U}} - \boldsymbol{U}_c)\cdot\frac{\mathsf{G}\boldsymbol{N}}{\|\mathsf{G}\boldsymbol{N}\|}\;\sigma_q \mathsf{G}\boldsymbol{N}\mathrm{d}L \\
&\simeq P_c(p)\int_{\partial\Omega_c\cap\partial\Omega_{pc}} \sigma_q \mathsf{G}\boldsymbol{N}\mathrm{d}L \\
&\quad - \int_{\partial\Omega_c\cap\partial\Omega_{pc}} Z_c\,(\boldsymbol{U}_p - \boldsymbol{U}_c(p))\cdot\frac{\mathsf{G}\boldsymbol{N}}{\|\mathsf{G}\boldsymbol{N}\|}\;\sigma_q \mathsf{G}\boldsymbol{N}\mathrm{d}L
\end{aligned}
$$

- finally, $\boldsymbol{F}_{pc}^{q}$ writes

$$
\boldsymbol{F}_{pc}^{q} = P_c(p)\, l_{pc}^{q}\boldsymbol{n}_{pc}^{q} - \mathsf{M}_{pc}^{q}\,(\boldsymbol{U}_p - \boldsymbol{U}_c(p)) \qquad (19)
$$

- M_{pc}^q are defined as $\mathrm{M}_{pc}^q = Z_c \int_{\partial\Omega_c \cap \partial\Omega_{pc}} \frac{\mathsf{G}\boldsymbol{N}}{\|\mathsf{G}\boldsymbol{N}\|} \otimes \mathsf{G}\boldsymbol{N}\, \sigma_q \mathrm{d}L$

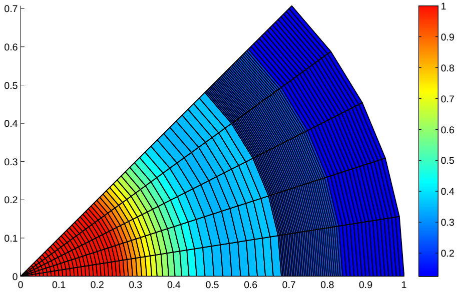


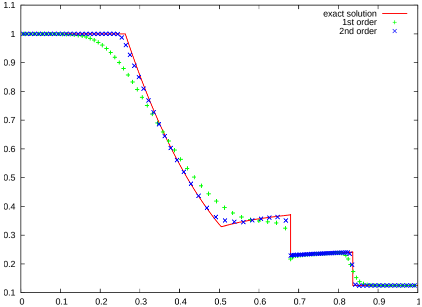
$$= Z_c\,(l_{pc}^{q,+}\,\boldsymbol{n}_{pc}^+ \otimes \boldsymbol{n}_{pc}^+ + l_{pc}^{q,-}\,\boldsymbol{n}_{pc}^- \otimes \boldsymbol{n}_{pc}^-)$$

where $l_{pc}^{q,\pm} = \int_{\partial\Omega_c \cap \partial\Omega_{pc}^\pm} \sigma_q \mathrm{d}L$

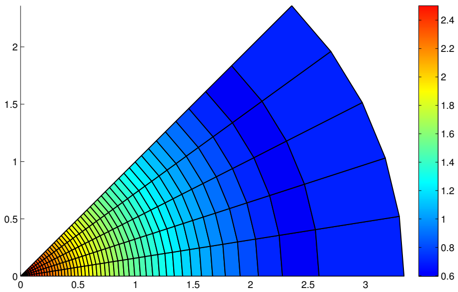
- $\mathrm{M}_{pc}^0 = \mathrm{M}_{pc} = Z_c\,(l_{pc}^+ \boldsymbol{n}_{pc}^+ \otimes \boldsymbol{n}_{pc}^+ + l_{pc}^- \boldsymbol{n}_{pc}^- \otimes \boldsymbol{n}_{pc}^-)$ is semi definite positive matrix with a physical dimension of a density times a velocity
- to be conservative in total energy over the whole domain, $\sum_{c \in \mathcal{C}(p)} \boldsymbol{F}_{pc} = \mathbf{0}$ and consequently

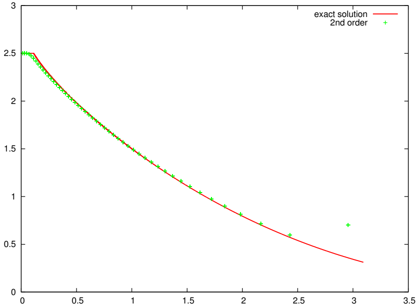
$$\Big(\sum_{c \in \mathcal{C}(p)} M_{pc}\Big)\,\boldsymbol{U}_p = \sum_{c \in \mathcal{C}(p)} [P_c(p)\, l_{pc}\boldsymbol{n}_{pc} + \mathrm{M}_{pc}\,\boldsymbol{U}_c(p)] \qquad (20)$$



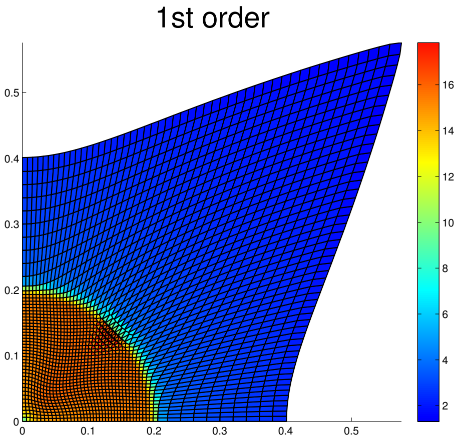


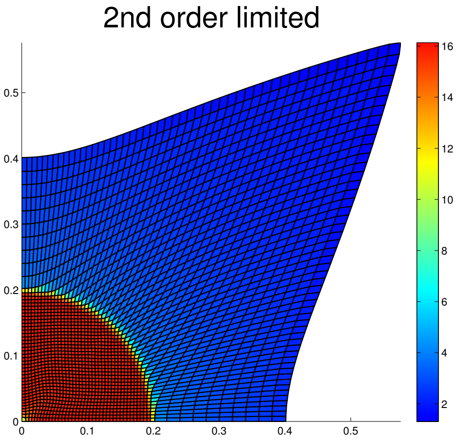
Sod shock tube problem on a polar grid made of 500 cells: density map with limitation



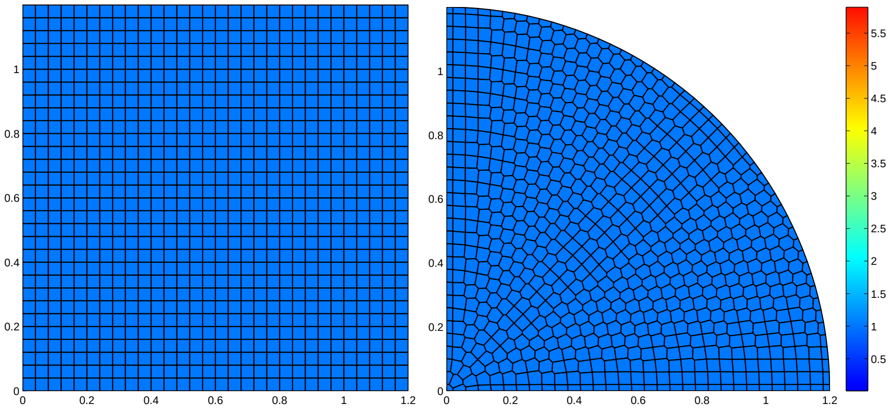


expansion wave into vacuum problem on a polar grid made of 250 cells: internal energy map with limitation

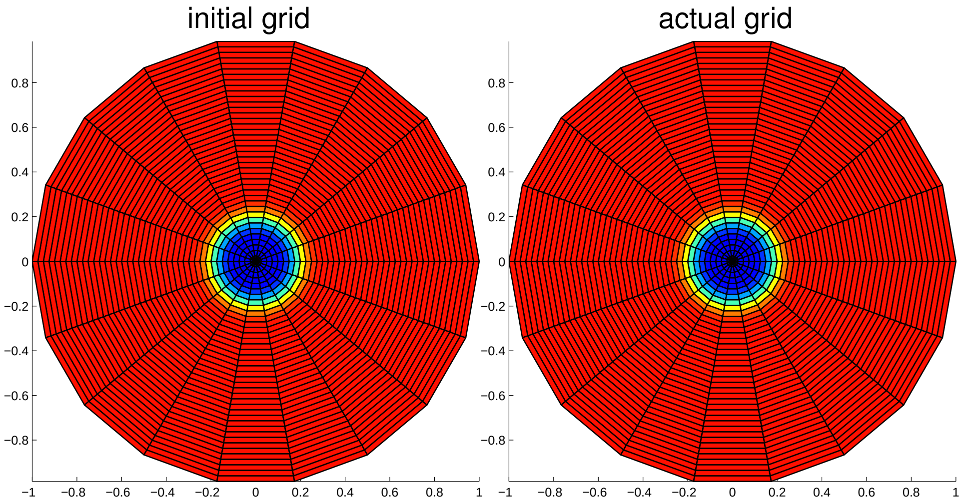




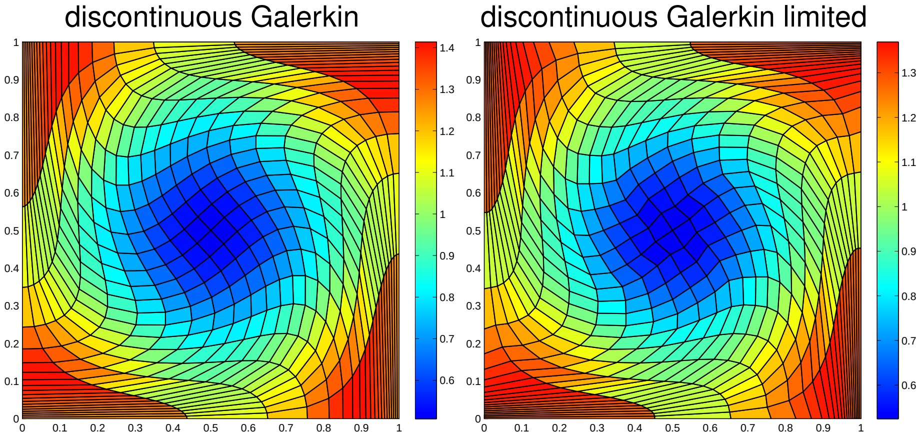
Noh problem on a Cartesian grid made of 2500 cells: density map



Sedov problem on a Cartesian grid made of 900 cells and a polygonal one made of 775 cells: density map with limitation



Gresho problem on a polar grid made of 720 cells: pressure map with limitation



Taylor-Green vortex problem on a cartesian grid made of 400 cells: pressure map without limitation at t=0.75s

	without limitation		with limitation	
h	$q^h_{L_2}$	$q^h_{L_\infty}$	$q^h_{L_2}$	$q^h_{L_\infty}$
$\frac{1}{20}$	1.74	1.35	2.05	1.54
$\frac{1}{40}$	1.85	1.85	2.11	1.81
$\frac{1}{80}$	1.42	2.34	1.58	1.54

Table: rate of convergence computed for second order DG scheme

	Green Muscl		Discontinuous Galerkin	
h	$E^h_{L_2}$	$E^h_{L_\infty}$	$E^h_{L_2}$	$E^h_{L_\infty}$
$\frac{1}{20}$	1.854E-2	6.596E-2	1.120E-2	3.678E-2
$\frac{1}{40}$	6.500E-3	2.452E-2	3.356E-3	1.446E-2
$\frac{1}{80}$	1.817E-3	9.122E-3	9.314E-4	4.019E-3
$\frac{1}{160}$	4.944E-4	2.555E-3	3.471E-4	7.959E-4

Table: numerical errors computed at t=0.6s on the pressure

1 Introduction
- Discontinuous Galerkin (DG)
- Scalar conservation laws
- 1D Lagrangian hydrodynamics

2 2D Lagrangian hydrodynamics
- References
- System and equations
- Geometric consideration
- 2nd order Deformation tensor
- 2nd order DG scheme

3 Conclusion

Conclusions

- DG schemes up to 3rd order
  - linear and non-linear scalar conservation laws in 1D and 2D on general unstructured grids
  - 1D gas dynamics system in Lagrangian formalism
- DG scheme up to 2nd order for the 2D gas dynamics system in Lagrangian formalism with particular geometric consideration
- numerical flux studies
- Riemann invariants limitation

Prospects

- 3rd order DG scheme for the 2D gas dynamics system in Lagrangian formalism
- validation
- extension to ALE